DIAS Delhi Institute for Administrative Services
India's Leading Institute for Civil Services Examination

ALL INDIA TEST SERIES CSE-2023

Candidate 's Information

1. NAME:- ....VIBHOR....BHARDWAJ......
2. UPSC ROLL NO:- ..0850373...........
3. MOBILE NO:- ......[redacted]......
4. SUBJECT:- ....Physics...Paper-II
5. DATE:-.....17 July 2023

FOR OFFICE USE ONLY:-

Q.NO	MARKS
1.	26½
2.	29
3.	23½
4.	24
5.	
6.	29
7.	
8.	

TOTAL MARKS	142/250

Good

EXAMINER SIGNATURE

INVIGILATOR SIGNATURE

18 Pusa Road Karol Bagh New Delhi-110005 Ph:01140079000, 9350934622 e-mail diasindia123@gmail.com

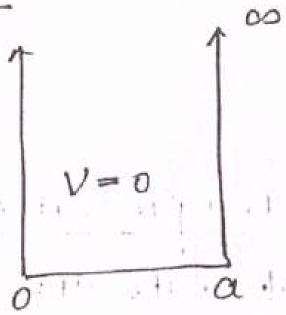
Section: A

Ans 1(a) Bohr's correspondence principle states that any physical system in quantum state starts to behave like classical system when the quantum numbers start approaching greater values.

Electron in a potential box:

Energy of e^- in box

$$\boxed{E_n = \frac{n^2\pi^2\hbar^2}{2ma^2}} \longrightarrow (1)$$



classically the energy of electron

$$E = \frac{1}{2}mv^2 = \frac{p^2}{2m} \longrightarrow (2)$$

Now for $p = \hbar K$

$$\Rightarrow p^2 = \hbar^2 K^2$$

or for 1-D potential box allowed energy values and hence K-values are

$$K = \frac{n\pi}{a}$$

So $p^2 = \hbar^2\left(\frac{n\pi}{a}\right)^2 = \frac{n^2\pi^2\hbar^2}{a^2}$

$$\Rightarrow \quad p^2 = \frac{\hbar^2 n^2 \pi^2}{a^2}$$

$$\text{or} \quad \frac{p^2}{2m} = \frac{\hbar^2 \pi^2 n^2}{2ma^2} \longrightarrow (3)$$

3/10

from (1) and (3)

$$\boxed{E_n = \frac{P_n^2}{2m}}$$ Hence proved.

Ans 1(b) de Broglie's matter wave: de Broglie hypothesized that like the nature of waves behaving like particles (eg. em waves), the particles motion can too be associated with waves.

(OR)

Moving particles have wave nature associated with them which possess wavelength:

$$\boxed{\lambda = \frac{h}{p}}$$ with $h \rightarrow$ Planck's constant
$p \rightarrow$ particle momentum

प्रश्न संख्या (Question No.)

DIAS

इस भाग में कुछ न लिखें (Don't write anything in this part.)

(i) e^- accelerated through 10 volt:

energy $E = qV = 1.6\times10^{-19}\times 10$
$= 16\times10^{-19}$ Joules

$\because$ $E = 10eV < 0.511\ MeV$ so Non-relativistic case will be considered: for which:

$$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.6\times10^{-34}}{(2\times9.1\times10^{-31}\times16\times10^{-19})^{\frac{1}{2}}}$$

$= 0.38\times10^{-9}m$

$\boxed{\lambda = 3.800\ \mathring{A}}$

3/5

(ii) $KE = 1\ MeV$ $(\because V = 1MV)$

$E > E_0 (= 0.511\ MeV) \Rightarrow$ Relativistic Case.

so $E^2 = p^2c^2 + m_0^2c^4$

or $(pc)^2 = E^2 - (m_0c^2)^2$ $(E = KE + m_0c^2)$
$= (1.51)^2 - (0.51)^2$ $= 1 + 0.511$ $= (1.511)$

$(pc)^2 = (2.0\cancel{9}\cancel{40})$

$\boxed{PC = \cancel{0.86}\ 1.42\ MeV}$ or $p = 0.86\ \frac{MeV}{c}$

2/5

5/10

$$\lambda = \frac{h}{p} = \frac{6.6\times10^{-34}}{\cancel{0.86}\ 1.42\times10^6\times1.6\times10^{-19}}\times3\times10^8 = \cancel{14.38}\ 8.71\times10^{-13}m.$$

$1.42\times10^{-2}\ \mathring{A}$

$$\boxed{\lambda = 0.008710\ \text{Å}}$$

Hence e^- behaves as a wave travelling with de-Broglie wavelength.

A: 1 (c)

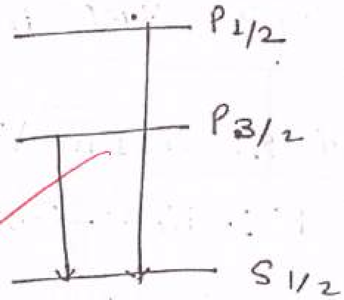
Zeeman effect is splitting of spectral line in presence of a WEAK magnetic field.

Calcium: $\lambda = 4226.73\ \text{Å}$.

$B = 4\ \text{wb}/m^2$.

for $P \rightarrow S$

for Normal zeeman effect



Energy $E = g\, m_j\, \mu_B B$.

$$\Delta E = \mu_B B.$$

$$\Delta E = 4 \times \frac{e\hbar}{2m}$$

$$\because\ E = \frac{hc}{\lambda} \Rightarrow \Delta E = \frac{hc}{\lambda^2}\Delta\lambda.$$

$$|\Delta\lambda| = \frac{\lambda^2}{hc}\Delta E = \frac{\lambda^2}{\cancel{h}c} \times \frac{\cancel{4} \times e\cancel{h}}{\cancel{2}\pi \cdot \cancel{2}m}$$

$$\boxed{\Delta\lambda = \frac{\lambda^2 e}{m\pi c}.}$$

$$\Delta\lambda = \frac{(4226.73)^2 \times 10^{-20} \times 1.6\times10^{-19}}{9.1\times10^{-31} \times 3.14 \times 3\times10^{8}} \ m.$$

$$= 333454.5 \times 10^{-16} \ m$$

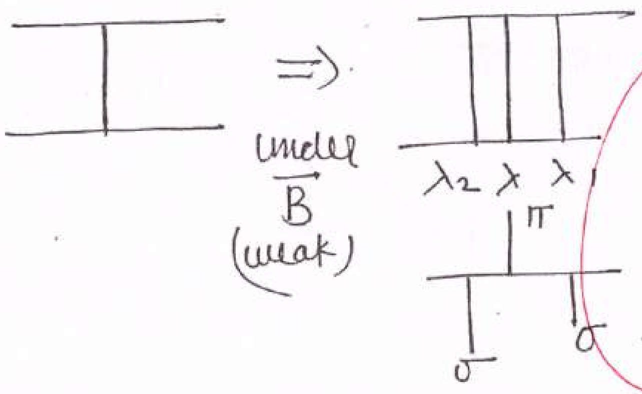
$$\boxed{\lambda = 0.33\ \text{Å}}$$

So wavelength of 3 components will be.

(i) Unaffected: $\lambda = 4226.73$ Å

(ii) Increased $\lambda_1 = \lambda + \Delta\lambda = 4227.06$ Å

(iii) decreased $\lambda_2 = \lambda - \Delta\lambda = 4226.4$ Å



6/10

Ans 1 (d) (i) Expectation value of a physical quantity is the average value of that quantity over a range of wavefunction value in which there is a probability for that wavefunction to exist. It is defined as

$$\boxed{\langle x \rangle = \frac{\int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi(x)\, dx}{\int_{-\infty}^{\infty} \psi^*(x)\psi(x)\, dx}}$$

Help in information: with wavefunction spanning the entire space, it is not possible to know the exact value of physical quantity. Hence expectation value provides an average over the entire range of wavefunction.

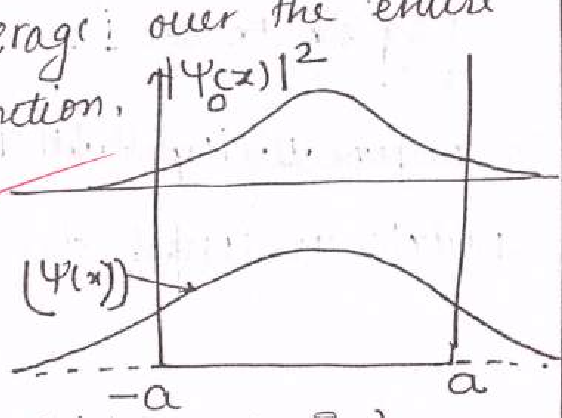
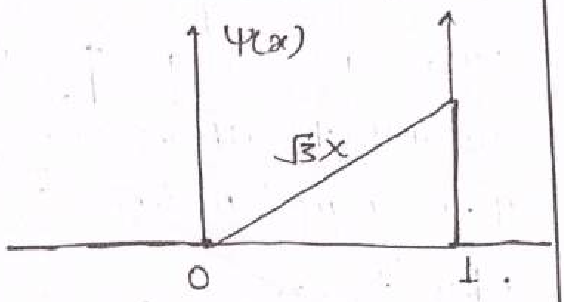


Fig: Finite Square potential well (groundstate wavefunction)

(ii) $\psi(x) = ax \qquad 0 < x < 1$

$\quad = 0 \qquad$ elsewhere.

Normalizing the wavefunction:



$$\int_{-\infty}^{\infty} |\psi(x)|^2 dx = 1.$$

$$\int_0^1 a^2 x^2 dx = 1 \Rightarrow a^2 \frac{x^3}{3}\Big|_0^1 = 1.$$

$$\frac{a^2}{3} = 1 \Rightarrow \boxed{a = \sqrt{3}}$$

So, $\boxed{\begin{aligned}\psi(x) &= \sqrt{3}x \quad , \quad 0 < x < 1 \\ &= 0 \qquad \text{elsewhere}\end{aligned}}$

Probability of finding particle between $x = 0$ to 1

$$\int_0^1 3x^2 dx = 1.$$

So probability will be UNITY cos particle is confined in between 0 & L

6/10

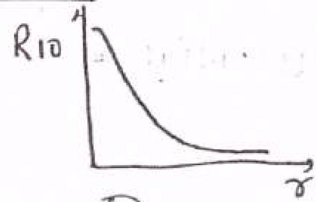
Ans (1)e:

Hydrogen atom, the simplest quantum mechanical system has ground state wavefunction:

$$\psi_{100} = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}, \quad a_0 = \frac{\hbar^2}{me^2}$$

Potential energy of e^- in H-atom:

$$V = \frac{-e^2}{4\pi\epsilon_0 r}$$



$$\text{So} \quad \langle V \rangle = \frac{-e^2}{4\pi\epsilon_0} \left\langle \frac{1}{r} \right\rangle \longrightarrow (1)$$

$$\& \left\langle \frac{1}{r} \right\rangle = \frac{\int_r \int_\theta \int_\phi \psi_{100}^* \cdot \frac{1}{r} \cdot \psi_{100}\, r^2 \sin\theta\, dr\, d\theta\, d\phi}{\int_r \int_\theta \int_\phi \psi_{100}^* \cdot \psi_{100}\, r^2 dr \sin\theta\, d\theta\, d\phi}$$

as the given ψ_{100} is already normalized.

$$\int (\psi^*) \psi \, d\tau = 1.$$

$$\text{so} \left\langle \frac{1}{r} \right\rangle = \frac{4\pi}{\pi a_0^3} \int_0^\infty e^{-2r/a_0}\, r \cdot dr$$

$$= \frac{4}{a_0^3} \left[\frac{\sqrt{2}}{\left(\frac{2}{a_0}\right)^2} \right] = \frac{1}{a_0}.$$

6½/10

so from eq^n - 1 :

$$\frac{-e^2}{4\pi\epsilon_0}\left\langle \frac{1}{r} \right\rangle = \boxed{\langle V \rangle = \frac{-e^2}{4\pi\epsilon_0 a_0}}$$. Ans

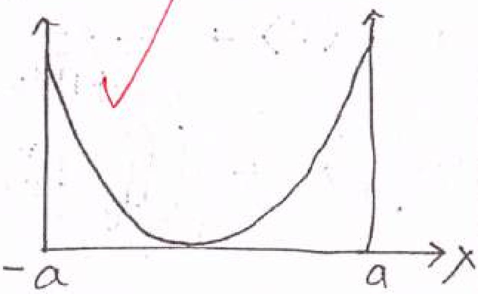
Question: 3

Ans 3(a) Particle in Parabolic Potential well : (x-direction)

Potential = $V = \frac{1}{2} m\omega^2 x^2$ for $-a<x<a$

$= 0$ elsewhere

Hamiltonian of such potential trapped particle



$$H = \frac{p^2}{2m} + V$$

$$= \frac{p_x^2}{2m} + \frac{1}{2} m\omega^2 x^2 = \frac{1}{2}\hbar\omega\left(\frac{p_x^2}{m\hbar\omega} + \frac{m\omega}{\hbar} x^2\right)$$

$$\boxed{H = \frac{1}{2}\hbar\omega\left(\hat{q}^2 + \hat{p}^2\right)} \longrightarrow (1)$$ with $\hat{p}^2 = \frac{p_x^2}{m\hbar\omega}$, $q^2 = \frac{m\omega}{\hbar}\hat{x}^2$

let two operators ;

$a = \frac{1}{\sqrt{2}}(q + ip)$ Lowering operator

$a^{\dagger} = \frac{1}{\sqrt{2}}(q - ip)$ Raising operator

Now $a^\dagger a = \frac{1}{2}(\hat{q}^2 + \hat{p}^2 + i(qp - pq))$

$$2a^\dagger a = q^2 + p^2 + i[q, p]. \longrightarrow (2)$$

$$[q, p] = \sqrt{\frac{m\omega}{\hbar}} \cdot \sqrt{\frac{1}{m\omega\hbar}} \cdot [\hat{x}, p_x] = \frac{i\hbar}{\hbar} = i.$$

$$\Rightarrow \boxed{q^2 + p^2 = 1 + 2a^\dagger a}$$

Putting in (1) $\boxed{H = \frac{1}{2}\hbar\omega(2a^\dagger a + 1)}$ —(3)

Defining $a^\dagger a = N =$ Number operator:

$$\boxed{H = \frac{\hbar\omega}{2}(2N+1)} \longrightarrow (4)$$

Now $\because$ $[\hat{N}, H] = 0$ $\Rightarrow$ N and H will have a joint eigen state $|n\rangle$ such that

$$\hat{N}|n\rangle = n|n\rangle$$

$$H|n\rangle = E_n|n\rangle.$$

or $(2N+1)\frac{\hbar\omega}{2}|n\rangle = E_n|n\rangle$ from (3)

$$\hbar\omega(2N|n\rangle) + \frac{\hbar\omega}{2}|n\rangle = E_n|n\rangle$$

$$\boxed{(2n+1)\frac{\hbar\omega}{2} = E_n}$$

So energy eigen value is

$$\therefore \boxed{E_n = \left(n+\frac{1}{2}\right)\hbar\omega}$$

Wave function: for ground state wavefunction $|0\rangle$

first excited state $|1\rangle = a^{+}|0\rangle$

2nd " " $|2\rangle = \frac{a^{+}}{2!}|1\rangle = \frac{(a^{+})^2}{2!}|0\rangle$

$\vdots$

n^{th} " " $|n\rangle = \frac{1}{n!}(a^{+})^n|0\rangle$

$$\boxed{\Psi_n = \frac{1}{n!}(a^{+})^n|0\rangle} \longrightarrow (5)$$

$$\because a^{+} = \frac{1}{\sqrt{2}}\left(\sqrt{\frac{m\omega}{\hbar}}\,x^2 - i\frac{p_x^{\,2}}{m\hbar\omega}\right).$$

$$a = \frac{1}{\sqrt{2}}\left(\frac{m\omega}{\hbar}x^2 + \frac{1}{m\hbar\omega}p_x^{\,2}\right)$$

$$= \frac{1}{\sqrt{2}}\left[\left(\sqrt{\frac{m\omega}{\hbar}}\right)^2 x^2 + \frac{1}{m\hbar\omega}\left(-i\hbar\frac{d}{dx}\right)^2\right]$$

$$\boxed{a = \frac{1}{\sqrt{2}}\left[\frac{m\omega}{\hbar}x^2 - \frac{\hbar}{m\omega}\frac{d^2}{dx^2}\right].}$$

$$a = \frac{1}{\sqrt{2}}\left[\frac{x^2}{x_0} - x_0\frac{d^2}{dx^2}\right] = \frac{1}{\sqrt{2}x_0}\left[x^2 - x_0^2\frac{d^2}{dx^2}\right]$$

$$a^{+} = \frac{1}{\sqrt{2}x_0}\left[x^2 + x_0^2\frac{d^2}{dx^2}\right).$$

Using a for $\langle x|a|0\rangle = 0$.

or $\langle x|\frac{1}{\sqrt{2}x_0}\left(x^2 - x_0^2\frac{d^2}{dx^2}\right)|0\rangle = 0$.

$$\boxed{\psi_0(x) = \frac{1}{\sqrt{\sqrt{\pi}\cdot 2}}\cdot e^{-x^2/2x_0^2}}$$

& applying a^+ for $\langle x|a^+|0\rangle = \psi_1(x)$

we can find it too.

In general after using these iterations

generalized:

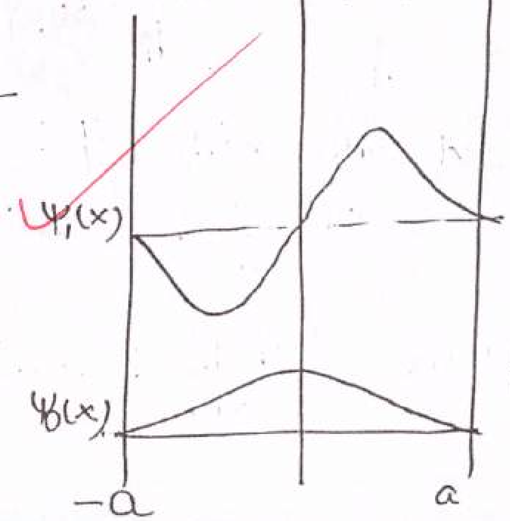
$$\psi_n(x) = \frac{1}{\sqrt{2^n\sqrt{\pi}\,n!}}\, e^{-x^2/2x_0^2}\cdot H_n\left(\frac{x}{x_0}\right)$$

12/20

where $H_n\left(\frac{x}{x_0}\right)$ are Hermite polynomials

$H_0\left(\frac{x}{x_0}\right) = 1$

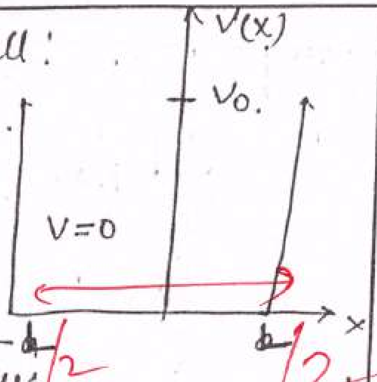
$H_1\left(\frac{x}{x_0}\right) = 2\left(\frac{x}{x_0}\right)^2$



Ans. 3(b)

Finite square potential well:

$$V(x) = \begin{cases} 0 & |x| < a \\ V_0 & |x| > a \end{cases}$$



The transcedental equations are:

$$\boxed{\begin{aligned} \eta &= \xi \tan \xi \\ \eta &= -\xi \cot \xi \end{aligned}}$$ where $\xi = K_2 a = \left[\frac{2m(V_0 - E)}{\hbar^2}\right]^{\frac{1}{2}} a$

$\eta = K_1 a = \frac{2mE}{\hbar^2} a$.

For bound states:

$$\eta^2 + \xi^2 = \gamma^2 \Rightarrow \left(\frac{2mV_0}{\hbar^2}\right) a^2 = \gamma^2$$

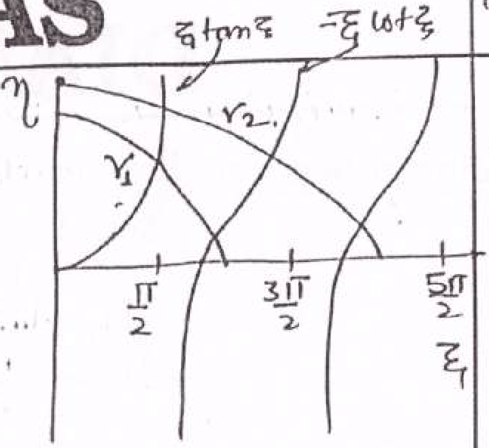
or for $a = L$

$$\boxed{\gamma^2 = \frac{2mV_0}{\hbar^2} L^2} = \frac{mV_0 L^2}{2\hbar^2}$$ Strength parameter

Bound States exits if:

$$(N-1)\frac{\pi}{2} < \gamma < N\pi \quad = N \text{ bound states}$$

given: $\frac{\pi^2 \hbar^2}{8m} < V_0 L^2 \leq 4\frac{\pi^2 \hbar^2}{2m}$



$$\frac{\pi^2}{4} \leq \frac{2mV_0}{\hbar^2} L^2 \leq 4\pi^2$$

$$\text{or } \left(\frac{\pi}{2}\right)^2 \leq \gamma^2 \leq (2\pi)^2$$

$$\boxed{\frac{\pi}{2} \leq \gamma \leq 2\pi}$$

Becaus γ lies between $\frac{\pi}{2}$ and 2π or we can say that it lies between 90° & 360°.

8/20

for $\frac{3\pi}{2} < \gamma < 2\pi$ we know that total bound states : = 4 states as.

$0 - \frac{\pi}{2}$	1
$\frac{\pi}{2} - \pi$	2
$\pi - \frac{3\pi}{2}$	3
$\frac{3\pi}{2} - 2\pi$	4

Bound states for different γ-values.

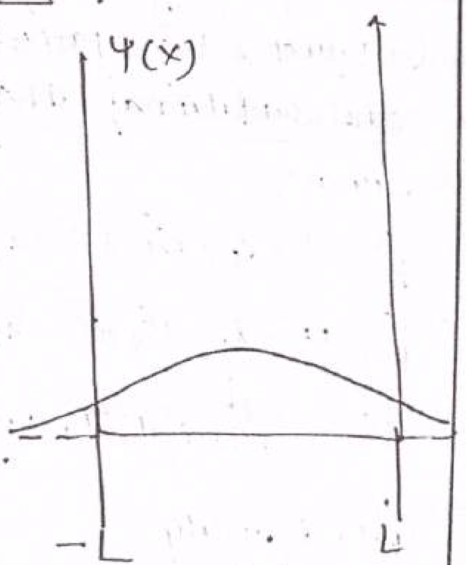
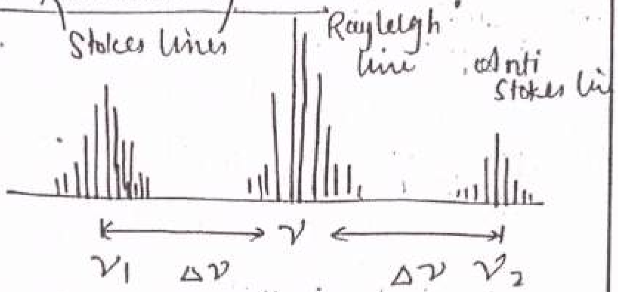
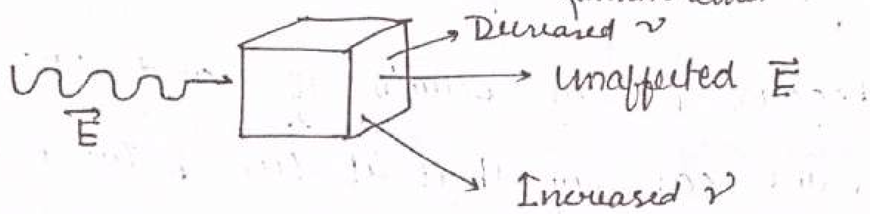


Fig: finitesquare well potential

Ans 3(c) Polarization in Raman spectra:





Method of polarization: Incident visible or UV ~~spectral~~ radiation causes the induced polarization in material (liquid, transparent solid, gas) which causes the rotational and vibrational polarization such as:

$$\vec{p}_v = \alpha_{0v}\vec{E_0} + \alpha_{\perp v}\vec{E_0}\sin\omega' t\,[\sin \underset{\text{Oscillation freq}}{\omega_0} t].$$

$$\vec{p}_r = \alpha_{0r}\vec{E_0} + \alpha_{\perp r}\vec{E_0}\sin\omega' t\,[\sin 2\omega_r t]$$

$\vec{p}$ → Polarizability

α_{0r} → Polarizability constant.

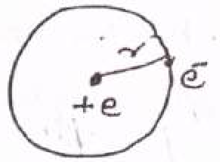
ω_r → Rotational frequency

3/10

and hence based on the material different degree of plane polarization can be found even though the incident radiation is unpolarized.

Question 2

Ans 2(a) **Hydrogen atom**



Hydrogen atom is the simplest atom with one electron and one proton.

Time independent Schrödinger equation for such system:

$$\nabla^2 \psi(r\theta\phi) + \frac{2m(E-V)}{\hbar^2}\psi(r,\theta,\phi) = 0.$$

In spherical Polar coordinates:

$$\left[\frac{1}{r^2}\left(\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2}{\partial\phi^2}\right]\psi + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

let $\psi = R(r)\,Y(\theta,\phi)$ putting in eqn. and dividing by $R(r)\,Y(\theta,\phi)$

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{1}{Y}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] + \frac{2m}{\hbar^2}(E-V)r^2 = 0.$$

as 2nd term only depends on (θ,ϕ) and first & last on 'r' only so separating and equating to a constant. $l(l+1)$.

DIAS

Radial eq^n.

$$\frac{1}{R}\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) + \frac{2m}{\hbar^2}(E-V)r^2 = l(l+1)$$

let $R = \frac{u}{r}$ $\therefore$ $\frac{\partial}{\partial r}\left(r^2\frac{\partial R}{\partial r}\right) =$

$$\frac{\partial}{\partial r}\left(r^2\left\{-\frac{u}{r^2}+\frac{1}{r}\frac{du}{dr}\right\}\right) = \frac{d}{dr}\left(-u+r\frac{du}{dr}\right)$$

$$= -\frac{du}{dr} + r\frac{d^2u}{dr^2} + \frac{du}{dr} = r\frac{d^2u}{dr^2}$$

$$\Rightarrow \frac{1}{R}\, r\frac{d^2u}{dr^2} + \frac{2m(E-V)r^2}{\hbar^2} = l(l+1)$$

$$\boxed{\frac{d^2u}{dr^2} + \left[\frac{2m(E-V)}{\hbar^2} - \frac{l(l+1)}{r^2}\right]u = 0.}$$

Puting $V = \frac{-e^2}{4\pi\epsilon_0 r}$

$$\frac{d^2u}{dr^2} + \left[\frac{2m}{\hbar^2}\cdot\frac{e^2}{4\pi\epsilon_0 r} - \frac{l(l+1)}{r^2}\right]u = -\frac{2mE}{\hbar^2}u.$$

or $$\frac{d^2u}{dr^2} = \left[\frac{l(l+1)}{r^2} - \frac{e^2m}{2\pi\epsilon_0\hbar^2 r} - \frac{2mE}{\hbar^2}\right]u$$

let $-\frac{2mE}{\hbar^2} = k^2$.

$$\frac{d^2u}{dr^2} = \left[\frac{l(l+1)}{r^2} - \frac{me^2}{2\pi\epsilon_0\hbar^2 r} + k^2\right]u$$

$$\frac{d^2u}{dr^2} = k^2u\left[\frac{l(l+1)}{(rk)^2} - \frac{me^2}{2\pi\epsilon_0\hbar^2(rk)k} + 1\right].$$

let $kr = \rho$. , $\rho_0 = \frac{me^2}{2\pi\epsilon_0\hbar^2 k}$.

$$\boxed{\frac{d^2u}{d\rho^2} = \left[1 + \frac{l(l+1)}{\rho^2} - \frac{\rho_0}{\rho}\right]u} \rightarrow ①$$

Solutions to this equation :

(i) Asymptotic one –

$\rho \rightarrow 0$ $\quad \frac{d^2u}{d\rho^2} = \frac{l(l+1)}{\rho^2} \Rightarrow u = C\rho^{l+1} + D\rho^{-l}$

$$\boxed{u = D\rho^{l+1}} \quad \text{as } \rho^{-l} \text{ diverges}$$

$\rho \rightarrow \infty$ $\quad \frac{d^2u}{d\rho^2} = u \Rightarrow u = Ae^{-\rho} + Be^{\rho}$

or $\boxed{u = Ae^{-\rho}}$ as $\rho \rightarrow \infty$ B blows up

so $\quad u = \rho^{l+1} e^{-\rho} v(\rho)$

Putting in eq^n – ①

we get

$$\boxed{\rho \frac{d^2v}{d\rho^2} + 2(l+1-\rho)\frac{dv}{d\rho} + v(\rho_0 - 2(l+1)) = 0}$$

Solving it with $v = \sum_j c_j \rho^j$

we get $\quad c_{j+1} = \frac{2(j+l+1) - \rho_0}{(j+1)(j+2(l+1))} c_j$

for series to terminate

$c_{j_{max}+1} = 0$ or

$$2(j+l+1) = \rho_0$$

or $\boxed{\rho_0 = 2n}$

Now $k^2 = -\frac{2mE}{\hbar^2}$ and $\rho_0 = \frac{me^2}{2\pi\epsilon_0\hbar^2 k}$.

$$E = -\frac{\hbar^2 k^2}{2m} = -\frac{\hbar^2}{2m}\left(\frac{me^2}{2\pi\epsilon_0\hbar^2\rho_0}\right)^2$$

$$= \frac{-me^4}{8\pi^2\epsilon_0^2\hbar^2\rho_0^2}.$$

$$\boxed{E_n = \frac{-me^4}{2(4\pi\epsilon_0 n\hbar)^2}}$$

This is the value for energy eigenstates and for eigenfunction

$$\boxed{R_{nl}(r) = \frac{\rho^{l+1} e^{-\rho} v(\rho)}{r}}$$

Diagram

where $v(\rho)$ = functions given by above relation and also can be converted into Laguerre's polynomial.

Solving Azimuthal equation

$$\frac{1}{Y}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial Y}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2 Y}{\partial\phi^2}\right] = -l(l+1)$$

$\frac{11}{20}$

solve for $Y = \Theta(\theta)\,\Phi(\phi)$.

we get

$$Y = (-1)^m\left[\sqrt{\frac{(l-m)!\,(2l+1)}{(l+m)!\,4\pi}}\cdot P_l^m(\cos\theta)\, e^{im\phi}\right]$$

so complete wavefunction is

$$\psi_{nlm}(r\theta\phi) = R_{nl}(r)\, Y_{lm}(\theta,\phi)$$

$$\boxed{\psi_{nlm}(r,\theta,\phi) = \sqrt{\frac{(l-m)!\,(2l+1)}{(l+m)!\,4\pi}}\; \rho^{l+1} e^{-\rho} v(\rho)\, P_l^{-m}(\cos\theta)\, e^{im\phi} \quad \text{for } m>0.}$$

Ans 2(b) $\psi(\theta,\phi) = \frac{1}{\sqrt{5}} Y_1^{-1}(\theta,\phi) + \sqrt{\frac{3}{5}}\, Y_1^{0}(\theta,\phi) + \frac{1}{\sqrt{5}} Y_1^{1}(\theta,\phi)$

$$L_+|Y_{lm}\rangle = \sqrt{l(l+1)-m(m+1)}\; Y_{l,m+1}$$

$$L_+|\psi(\theta,\phi)\rangle = \frac{\hbar}{\sqrt{5}}\sqrt{2-0}\; Y_1^0 + \sqrt{\frac{3}{5}}\sqrt{2}\; Y_1^1 \hbar + 0 = \hbar\left(\sqrt{\frac{2}{5}}\, Y_1^0 + \sqrt{\frac{6}{5}}\, Y_1^1\right)$$

$$\langle\psi|L_+|\psi\rangle = \hbar\left(\frac{1}{\sqrt{5}} Y_1^{-1} + \sqrt{\frac{3}{5}} Y_1^0 + \frac{1}{\sqrt{5}} Y_1^1\right)\left(\sqrt{\frac{2}{5}}\, Y_1^0 + \sqrt{\frac{6}{5}}\, Y_1^1\right)$$

$$= \left[\frac{\sqrt{6}}{5}(Y_1^0 \cdot Y_1^0) + \frac{\sqrt{6}}{5} Y_1^1\right]\hbar$$

4/5

$$\boxed{\langle\psi|L_+|\psi\rangle = \frac{2\sqrt{6}\,\hbar}{5}}$$

(ii) Measuring L_z the values will be

$-\hbar, 0, \hbar$ for Y_1^{-1}, Y_1^0, Y_1^1.

after measuring L_z and finding it is

$-\hbar$ value then wavefunction will be

$$\psi = \frac{1}{\sqrt{5}} Y_1^{-1}(\theta, \phi)$$

so $L_x Y_{1,-1} = \frac{1}{2}(L_+ + L_-) Y_{1,-1}$

$$= \frac{1}{2}\left[L_+ Y_{1,-1} + L_- Y_{1,-1} \right]$$

$$= \frac{1}{2}\left[\sqrt{2}\, Y_{1,0} + 0 \right]\hbar = \frac{\hbar}{\sqrt{2}} Y_{1,0}$$

$\langle L_x \rangle = \langle Y_{1,-1} | L_x | Y_{1,-1} \rangle = 0.$

$$\boxed{\langle L_x \rangle = 0}$$

and $\langle L_y \rangle = \langle Y_{1,-1} | L_y | Y_{1,-1} \rangle$

$\therefore L_y | Y_{1,-1} \rangle = \frac{1}{2i}(L_+ - L_y) = 0.$

$$\boxed{(\langle L_y \rangle) = 0}$$

12/20

$\because \langle L_x^2 \rangle = \langle L_y^2 \rangle = \frac{\left[\langle L^2 \rangle - \langle L_z^2 \rangle\right]}{2}$

$$= \left[\frac{l(l+1) - m^2}{2}\right]\hbar^2$$

so $\langle L_x^2 \rangle = \langle L_y^2 \rangle = \left(\frac{2-1}{2}\right)\hbar^2 = \frac{\hbar^2}{2}.$

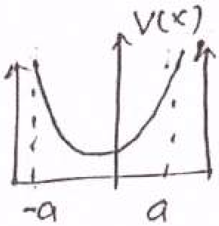
So $\Delta L_x = \sqrt{\langle L_x^2 \rangle - \langle L_x \rangle^2} = \frac{\hbar}{\sqrt{2}}$

$$\boxed{\Delta L_y = \frac{\hbar}{\sqrt{2}}}$$

So $\boxed{\Delta L_x = \frac{\hbar}{\sqrt{2}}, \quad \Delta L_y = \frac{\hbar}{\sqrt{2}}}$ Ans

Ans 2(c)

Harmonic Oscillator Energy



$$E = \frac{p_x^2}{2m} + \frac{1}{2}m\omega^2x^2$$

$$\Delta p_x \geq \frac{\hbar}{2\Delta x} \simeq \frac{\hbar}{2x} \quad \text{for} \quad \boxed{\text{Fig}}$$

$$E = \frac{\hbar^2}{8mx^2} + \frac{1}{2}m\omega^2x^2$$

for min. Energy $\left.\frac{dE}{dx}\right|_{x=x_0} = 0 \Rightarrow \frac{-2\hbar^2}{8mx_0^3} + m\omega^2x_0 = 0.$

$$\text{or} \quad \frac{2\hbar^2}{8mx_0^3} = m\omega^2x_0$$

$$x_0^4 = \frac{2\hbar^2}{8m^2\omega^2}$$

$$\boxed{x_0^2 = \frac{\hbar}{2m\omega}}$$

6/10

$$\text{or} \quad E_{min} = \frac{\hbar^2}{8m} \times \frac{2m\omega}{\hbar} + \frac{1}{2}m\omega^2 \times \frac{\hbar}{2m\omega}$$

$$= \frac{\hbar\omega}{4} + \frac{\hbar\omega}{4}$$

$$\boxed{E_{min} = \frac{\hbar\omega}{2}.}$$

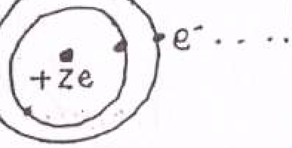
(Question No.)

DIAS

Section: B

Ans 5(a) Nucleus is a quantum mechanical system residing in the centre of an atom providing it the requisite mass and discovered via Rutherford scattering of α-particles

2 properties:



① CHARGE: Nucleus comprises of (i) Proton = total charge to nucleus.
(ii) Neutron = Neutral (No charge)

Hence Nucleus is positively charged.

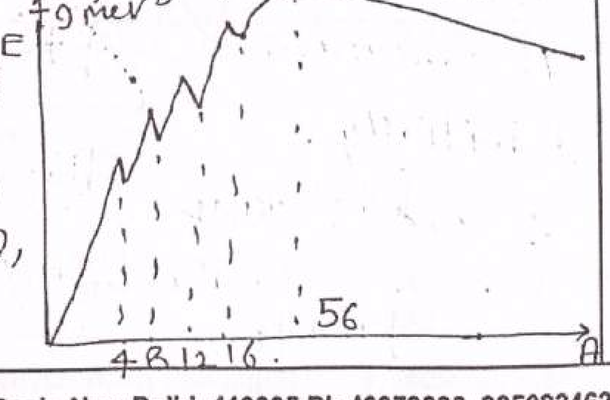
② Bindingenergy: It is energy which binds the nucleons inside nucleus together or the energy required to separate nucleons apart. Binding energy per nucleon is as per graph:-

Maximum for $\frac{BE}{A}$

A = 56

and discontinuous at A = 2, 8, 20, 28, 50, 86

(Magic Numbers)



Radius of $^{12}_{6}C$.

$$R = R_0 A^{1/3} = 1.2 \times (12)^{\frac{1}{3}} = \underline{2.74 \text{ fermi}}$$

$$\frac{\text{Atomic Radius}}{\text{Nuclear Radius}} \quad \frac{R_a}{R} = \frac{0.529 \times 10^{-10}}{2.74 \times 10^{-15}}$$

$$\boxed{R_a = 1.93 \times 10^{4} R}$$

6/10

So Atomic Radius is almost 20,000 times bigger than nuclear Radius. This shows nucleus is confined in very small region.

Ans 5(b)(i) $\boxed{\text{N-Z graph}}$

N v/s Z is plotted.

Inference –

As the Mass Number A increases:–

(i) Initially, Z and N overlap meaning, equal number of proton and Neutron

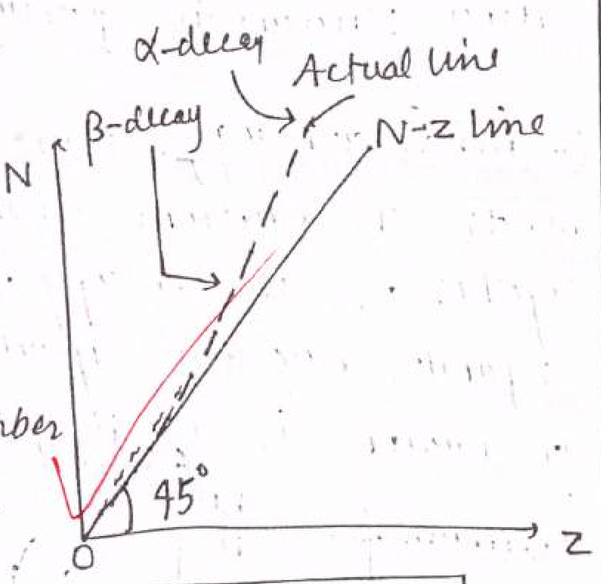


Fig- NZ plot

(ii) As further A increases; No. of Neutrons increases so as to COUNTER the Coulomb repulsion of protons (+e).

(iii) The actual line diverges from 45° N-Z line and in the upper portion Nuclei become UNSTABLE and they start decaying by α, β, γ radiation.

(ii) Nucleus stability:

(A) ${}^{7}_{3}Li$ = protons = 7 (odd)
Neutrons = 4. (even)

(B) ${}^{8}_{3}Li$ ⇒ protons = 3 (odd)
Neutrons = 5 (odd)

6/10

∵ Pairing energy E_p is zero for odd-even nuclei and is negative for odd-odd nuclei.

So from Semi-Empirical Mass formula:

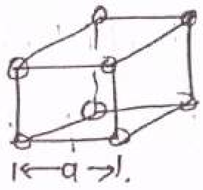
$$BE = a_v A - a_s A^{2/3} - a_c \frac{(Z)(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A}$$
$$(\pm, 0)\, a_p A^{-3/4}$$

we can say that

$$(BE)\Big|_{^7_3Li} > (BE)\Big|_{^8_3Li}$$

so $\boxed{^7_3Li \text{ will be more stable}}$

Ans 5(c) (i) $\boxed{\text{Simple Cubic}}$

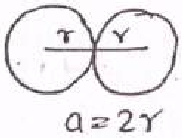


No. of atoms per unit cell = 8 × (atom shared between diff atoms) corners

$= 8 \times \frac{1}{8} = \boxed{1.}$

fraction volume occupied $= f = \frac{1 \times \frac{4}{3}\pi r^3}{a^3 \rightarrow \text{unit cell Volume}}$

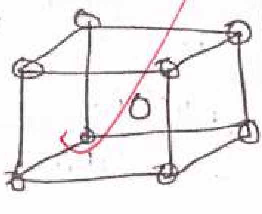
$= \frac{4}{3}\frac{\pi r^3}{8 r^3} = 0.52$



$a = 2r$

$\boxed{f = 0.52}$ or 52%.

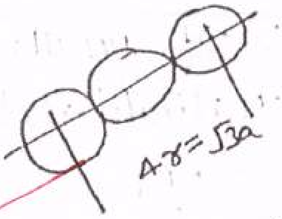
(ii) Body Centered



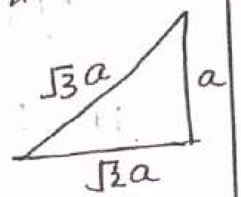
total atoms/unit cell $= 8 \times \frac{1}{8} + 1$

$= \underline{2}$.

$$f = \frac{2 \times \frac{4}{3}\pi r^3}{a^3}$$



$$= \frac{8\pi}{3} \frac{3\sqrt{3}\,a^3}{64}$$



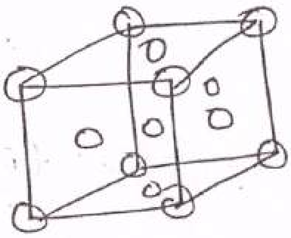
$\boxed{f = 0.67}$ or $\underline{67\%}$

(iii) $\boxed{fcc}$

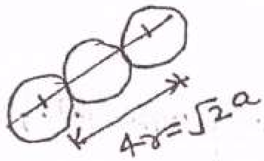
corner ↓ face ↓

$$\frac{\text{No. of atoms}}{\text{unit cell}} = \left(\frac{1}{8} \times 8\right) + 6 \times \frac{1}{2}$$

$$= \underline{4}$$



so $f = \dfrac{4 \times \frac{4}{3}\pi r^3}{a^3}$



$$= \frac{16\pi}{3} \times \frac{2\sqrt{2}\,a^3}{64\,a^3}$$

$\boxed{f = 0.74}$ or $\underline{74\%}$

Hence fcc is the most densely, closely packed structure in all 3.

6/10

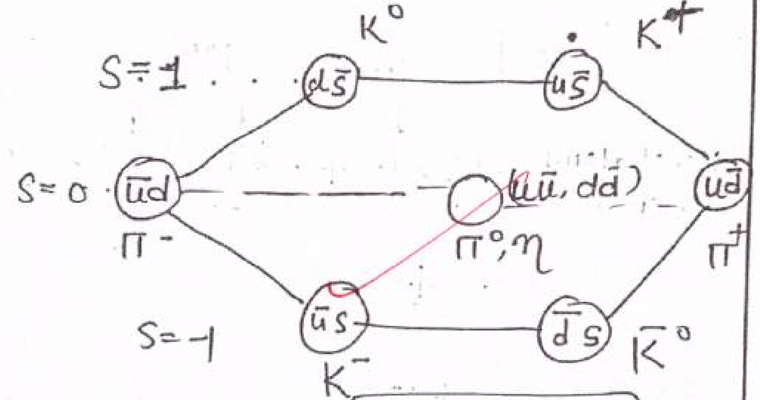
प्रश्न संख्या (Question No.)

DIAS

इस भाग में कुछ न लिखें (Don't write anything in this part.)

Ans 5(e) Quarks are the quasi-independent particles constituting the <u>HADRONS</u> (<u>Baryons</u> & <u>Mesons</u>)

(i) π^0 : Meson = Have ONE QUARK ONE ANTI-QUARK.



so from <u>Meson Octate</u> $\boxed{\text{Meson Octate}}$

$\boxed{\pi^0 : (u\bar{u} \text{ or } d\bar{d})}$ as Strangeness $= 0$.

(ii) K^+: from above diagram

$\boxed{K^+ = u\bar{s}}$ $\to Q = \frac{2}{3} + \frac{1}{3} = 1$, $\to S = +1$

6/10

(iii) Δ^{++} : Have $Q = +2$, spin $= \frac{3}{2}$. $S = 0$.

so $\boxed{\text{Quarks} = uuu}$ $\to q = 3 \times \frac{2}{3} = 2$, $\to S = 3 \times \frac{1}{2} = \frac{3}{2}$.

(iv) Σ^0 : from Baryon Octate:

$Q = 0$.
$S = -1$.

$$\boxed{\Sigma^0 \equiv uds}$$

Question 6

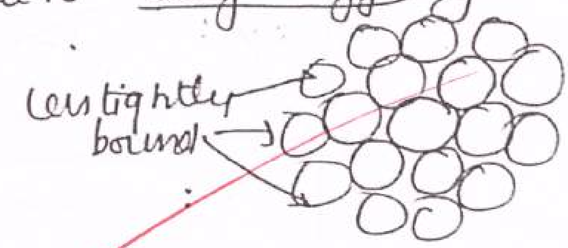
Ans 6(a) (i) Semi-Empirical Mass formula : given by Weizsacker its explain the Binding energy of nucleus.

$$B.E. = a_v A - a_s A^{2/3} - a_c \frac{Z(Z-1)}{A^{1/3}} - a_a \frac{(A-2Z)^2}{A} (\pm a_p, 0) A^{-3/4}$$

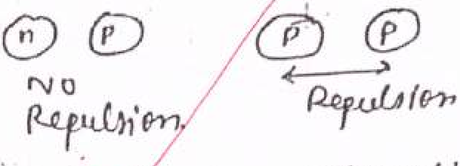
(i) Volume term – Signifies the constituent nucleons and their influence on binding energy it proportional to the Mass Number.

$$B_v = a_v A$$

(ii) Surface energy signifies the weakening of binding energy due to surface effect of nuclei.

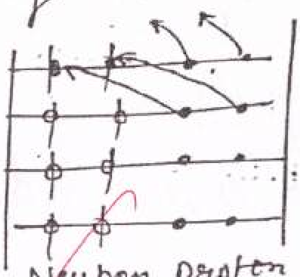


(iii) Coulomb Repulsion Energy is for PROTONS having charge "+e". This loosens the nucleus.



(iv) Antisymmetric energy – Due to differential number of neutrons and protons. Converting proton to the energy level of neutron requires :

$$E = \frac{\epsilon}{2} \frac{(N-2Z)^2}{A}$$

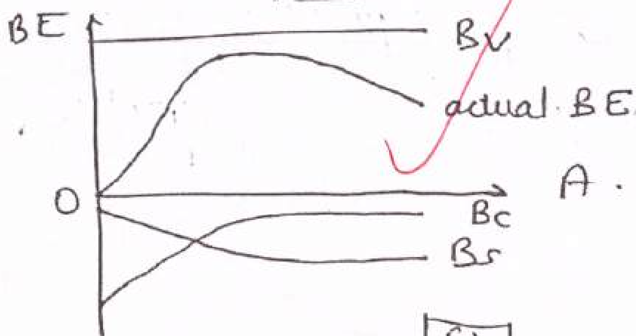


(v) Pairing energy:

Pairing between two nucleons create additional effect with:

$E_p =$ 0 for odd even
$+$ for even even
$-$ for even-odd nuclei. Hence

even-even provides for more binding and odd-odd loosens the nucleus.



Fig

(ii) Nuclear forces are manifestation of exchange of π-mesons as suggested by Mr. <u>YUKAWA</u>.

<u>Given</u>: $m_\pi = 270\, m_e$

range $r = c\Delta t$ (c = speed of π-meson $\cong$ speed of light)

$= \dfrac{c\hbar}{2\Delta E}$ $\left(\because \Delta E \Delta t \geq \dfrac{h}{2\pi}\right)$ $\geq \dfrac{\hbar}{2}$

$= \dfrac{c\hbar}{2mc^2}$

$r = \dfrac{\hbar}{2mc}$

$= \dfrac{1.05\times10^{-34}}{2\times 270\times 9.1\times10^{-31}\times 3\times10^{8}}$ m

$= 7.1225\times10^{-5}\times10^{-11}$

$= 7.12\times10^{-16}$ m.

$\boxed{r = 0.712 \text{ fm}}$

8/15

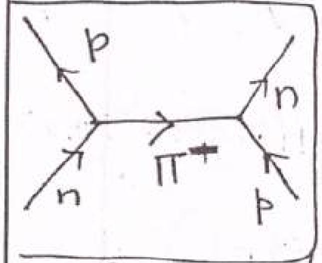


Fig- Feynmann diagram for Meson exchange

The uncertainity principle is not violated if ΔE is borrowed with $\dfrac{\hbar}{\Delta E}$ time.

so as to adhere to $\boxed{\Delta E \cdot \Delta t \geq \dfrac{\hbar}{2}}$

Ans 6(b) Shell Model of Nucleus

considering the 3-D potential for nuclei :

$$\left[\nabla^2 + \frac{2m(E-V)}{\hbar^2}\right]\psi = 0.$$

The energy values are given by -

$$E = \frac{\hbar^2 K^2}{2m}$$

$$\boxed{E_{nl} = \frac{\hbar^2 K_{nl}^2}{2m}}$$

which presents that there is a degeneracy in energy values.

For Harmonic Oscillator potential - $V = \frac{1}{2}m\omega^2 x^2$

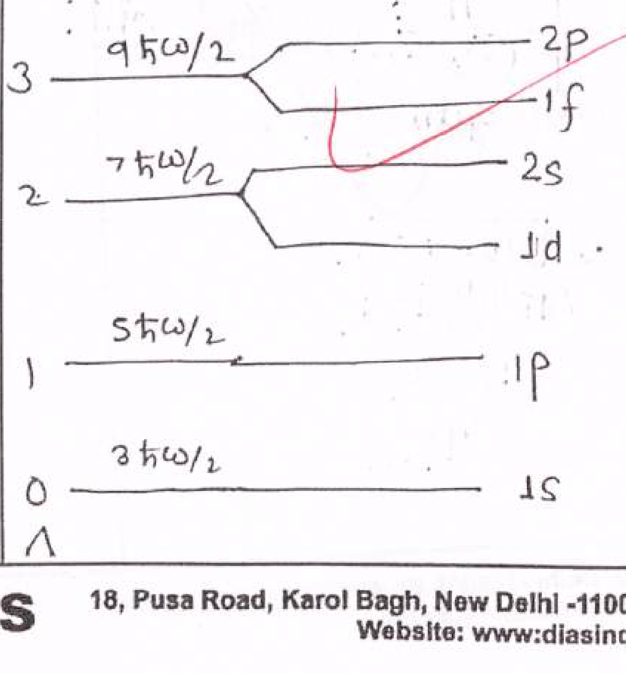
$$\boxed{E = \frac{\hbar\omega}{2}\left(\Lambda + \frac{3}{2}\right)} = \frac{\hbar\omega}{2}\left(2n + l - \frac{1}{2}\right)$$

Λ = Quantum Number

and which shows energy having large degeneracy.

Now to fill shells

λ	$\hbar\omega(\lambda+\frac{3}{2})$	nl	No. of nucleons	
0	$\frac{3}{2}\hbar\omega$	1s	2	2
1	$\frac{5}{2}\hbar\omega$	1p	6	8
2	$\frac{7}{2}\hbar\omega$	1d 2s	12	20
3	$\frac{9}{2}\hbar\omega$	1f 2p	20	40
4	$\frac{11}{2}\hbar\omega$	1g 2d 3s	30.	70
5	$\frac{13}{2}\hbar\omega$	1h 2f 3p	⋮	⋮
6	$\frac{15}{2}\hbar\omega$	1i 2g 3d	⋮	⋮
and..	so on			



Shells as per Harmonic Oscillator

9/15

Still there is degeracy which can be lifted up by considering SPIN-ORBIT coupling as suggested by Mrs. Mayer.

Ans 6(c)

Elementary Particles

- **Matter particles**
  - **Leptons**
    - electron
    - Muon
    - Tau
    - Neutrinos
    - ⇓ Spin-$\frac{1}{2}$
  - **Hadrons**
    - (1) **Baryons**
      - Nucleon (p, n)
      - Hyperons (Σ, Ξ, Λ)
      - Spin $= \frac{1}{2}, \frac{3}{2}, \ldots$
    - (2) **Mesons**
      - ($\pi^{\pm,0}, \bar{K}, K^0$, K^+, K^-) η etc
- **Quanta of Force**
  1. Photon ($S=1$)
  2. Gluons ($S=1$)
  3. $W^{\pm}$, Z Bosons ($S=1$)
  4. Graviton ($S=2$)
  - ⇓ Integral Spin

Interactions

① strong interactions:

(i) Having very short range $\sim 10^{-15}m$.

(ii) Characteristic time $\sim 10^{-23}$ sec (very very short)

(iii) Mesonic exchange is the reason

(iv) Quanta of force is Gluon which provides the requisite interaction among nucleons.

(v) All the conservation laws are followed.

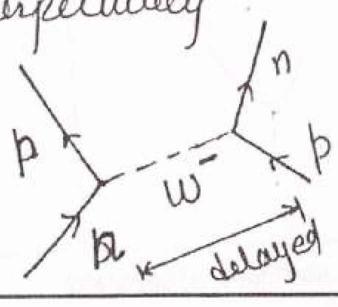
Example: $\boxed{p + \pi^- \longrightarrow \Lambda^0 + K^0}$

② weak interaction: There are –

(i) very very short range $\sim 10^{-17}m$.

(ii) Characteristic time is about 10^{-10} sec.

(iii) Force quanta are $W^\pm$, Z bosons having mass 81 GeV/c^2 and 90 GeV/c2 respectively.



(iv) Example is β-decay:

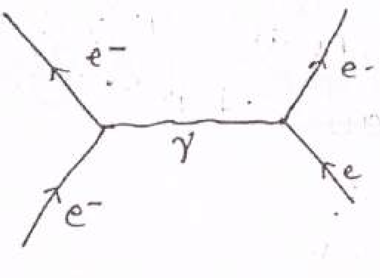
$$\boxed{n \longrightarrow p^+ + \beta^- + \bar{\nu}}$$

In weak interactions following laws are violated — (a) Parity conservation
(b) Isospin "
(c) Hypercharge "
(d) Strangeness "

(3) Electromagnetic interaction — with exchange of virtual photon having nearly zero mass, the range becomes infinite.

- Quanta of force = Photon (spin = 1) (virtual)
- Conservation of all except Isospin (I) is found.

12/20



Eg: $\boxed{e^+ + e^+ \longrightarrow \gamma + \gamma}$

(Question No.)

DIAS

(Don't write anything in this part.)

(4) Gravitational Interaction: It is assumed to be due to exchange of virtual gravitons having spin = 2 and range as infinite.

This interaction is the least studied. Efforts have been made to create a single Unified theory for all interactions. But with Grand Unified theory only 3 — strong, weak, em have been unified. String Theory and Quantum Loop theory are two working in this direction